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Pricing Catastrophic Risk under a Hybrid Reinsurance Treaty: Empirical Evidence from a Peaks-Over-Threshold Approach

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ABSTRACT

BACKGROUND AND OBJECTIVES: Catastrophic risk refers to the exposure of a large population or insurance portfolio to extreme, low-frequency yet high-severity events that can simultaneously generate substantial financial, human, and infrastructural losses. Such events exceed the volatility typically captured by conventional risk models and frequently challenge the solvency and underwriting capacity of insurance and reinsurance markets. Examples include large-scale earthquakes, floods, droughts, and acts of terrorism, all of which can severely disrupt national economies and necessitate external financial assistance for recovery. In the context of Iran—recognized as one of the most disaster-prone countries globally, particularly in terms of seismic activity—the management of catastrophe risk constitutes not only an actuarial necessity but also a public policy priority. Over the past three decades, more than 80,000 fatalities have been attributed to earthquakes alone, underscoring the urgent need for robust catastrophe risk modeling and scientifically grounded reinsurance pricing mechanisms. Traditional actuarial models that rely on limited historical observations or frequency–severity distributions often fail to capture the heavy-tailed nature of catastrophic losses. This limitation motivates the adoption of advanced statistical and probabilistic approaches, such as Extreme Value Theory (EVT), which provide a more accurate characterization of rare but devastating events. The primary objective of this study is to develop and empirically validate a pricing framework for catastrophe (CAT) reinsurance based on a hybrid treaty structure that integrates Quota Share (QS) and Excess of Loss (XL) arrangements. This hybrid design enables flexible allocation of both frequency and severity risks across multiple loss layers, thereby enhancing the efficiency and stability of catastrophe coverage.

METHODS: To quantify the tail behavior of catastrophic losses, earthquake fatality data for Iran covering the period 1950–2023 are analyzed using the Peaks Over Threshold (POT) method within the EVT framework. The POT approach focuses on exceedances above a specified high threshold, allowing precise modeling of extreme observations while minimizing bias from moderate losses. The Generalized Pareto Distribution (GPD), the canonical distribution under the POT framework, is employed to model excess fatalities. Its parameters are estimated using Maximum Likelihood Estimation (MLE), ensuring statistical efficiency and consistency under large-sample conditions. Based on the fitted GPD, tail probabilities and expected losses beyond the threshold are incorporated into a reinsurance pricing model grounded in the standard deviation premium principle. By accounting for both the expected value and the dispersion of losses, this principle provides a theoretically sound basis for risk loading and premium determination. The hybrid QS–XL structure is formally specified to capture the interaction between the retention level (□) and the quota share coefficient (□), thereby establishing a unified framework for calculating the reinsurer's expected indemnity and corresponding premium. In addition, sensitivity analyses are conducted across a range of retention levels and quota share ratios to evaluate the effects of contractual parameter variations on premium dynamics. The model is calibrated under realistic assumptions derived from Iranian earthquake data, reflecting both empirical severity patterns and practical treaty configurations commonly adopted by reinsurers operating in high-risk environments.

FINDINGS: The empirical results indicate a clear inverse relationship between the reinsurance premium and the retention level □. As the primary insurer retains a larger portion of the loss, the reinsured layer narrows, resulting in a lower reinsurance premium. Conversely, the premium exhibits a positive and approximately linear relationship with the QS coefficient (□); as the reinsurer's proportional participation increases, the total cost of coverage correspondingly rises. These findings confirm that catastrophe reinsurance pricing is highly sensitive to the structural parameters of the contract. For a fixed quota share proportion, higher retention levels substantially reduce premium costs. For a fixed retention level, increasing the reinsurer's proportional participation raises premiums due to the higher expected indemnity. This evidence highlights the inherent trade-off between risk transfer and premium affordability—a central consideration for actuaries and policymakers involved in disaster risk financing. Moreover, the POT-based tail modeling framework captures extreme quantiles of the loss distribution, providing more robust and transparent estimates of potential catastrophic payouts. Compared with conventional approaches that may underestimate tail risk, the EVT-based methodology enhances actuarial fairness and strengthens resilience against large and unpredictable claims. Supported by rigorous tail risk quantification, the hybrid QS–XL model offers a balanced contractual mechanism suited to both primary insurers and reinsurers seeking efficient coverage in highly exposed environments.

CONCLUSION: This study demonstrates that integrating EVT with hybrid reinsurance structures provides a rigorous actuarial framework for pricing catastrophic risk. The proposed QS–XL model effectively combines proportional and non-proportional components, ensuring both analytical tractability and operational flexibility. By grounding premium determination in statistical inference and long-term empirical data, the framework contributes to the financial stability of insurers and reinsurers operating in disaster-exposed regions.

In the case of Iran, where earthquake-related losses pose systemic threats to economic resilience, adoption of this hybrid model may enhance capital adequacy, promote equitable pricing of catastrophe coverage, and support sustainable disaster risk management. Beyond its national application, the proposed framework offers a replicable methodological approach for other emerging markets confronting similar challenges in estimating and managing catastrophic risk. Overall, the integration of EVT-based tail estimation with hybrid treaty design narrows the gap between statistical rigor and practical actuarial implementation, thereby contributing to the development of more resilient and data-driven catastrophe insurance systems.

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INTRODUCTION

A catastrophic risk is fundamentally characterized by the simultaneous exposure of a large population to the potential for significant financial or physical losses arising from a single high-severity peril. Such events commonly occur as natural hazards, including earthquakes, floods, or droughts, or as human-induced hazards such as large-scale terrorism, all of which can result in extensive loss of life and widespread infrastructure damage.

Iran is a disaster-prone country. According to global risk reports, Iran ranks among the countries with the highest vulnerability to natural hazards, where disasters have generated substantial financial, social, and physical losses. Over the past 30 years, more than 80,000 people have died directly as a result of natural disasters in Iran. Most provinces of the country are classified as arid or semi-arid regions. Throughout Iran's history, numerous natural disasters have occurred, including severe floods, extreme temperatures, and prolonged droughts. In addition, the country is highly susceptible to earthquakes and has experienced many devastating seismic events (Seddighi & Seddighi, 2020).

Within the insurance context, extreme events are defined by an exceptionally low probability of occurrence combined with the potential to generate severe financial losses for the insurance industry. When such low-frequency, high-severity perils materialize, the resulting losses exceed the capacity of a single insurer to absorb them independently. Consequently, the occurrence of these events may have catastrophic financial implications for individual underwriting entities.

Given the potential for catastrophic events in Iran, the effective management and control of catastrophe risk is of critical importance for insurance companies. Catastrophe (CAT) losses impose substantial financial pressure on primary insurers and may ultimately lead to insolvency. Accordingly, transferring CAT risk to reinsurers constitutes an essential risk-management strategy for ceding companies. This raises a fundamental question: what is the appropriate price of a CAT reinsurance contract?

The purpose of this study is to extend the CAT reinsurance premium model proposed by Chao (2022) to a hybrid QS–XL catastrophe reinsurance framework. The number of fatalities is used as the risk indicator for premium calculation. Extreme observations in the fatality data are modeled using the Peaks Over Threshold (POT) approach. This modeling choice is motivated by the high-severity nature of catastrophic events. The standard deviation premium principle is employed to determine the insurance premium. The remainder of this article is organized as follows. Section 2 presents a brief overview of Extreme Value Theory (EVT). Section 3 derives the catastrophe reinsurance pricing formulas for QS–XL CAT contracts. Section 4 provides an empirical application using a real dataset. Finally, Section 5 presents the conclusions and suggestions.

THEORETICAL BACKGROUND AND LITERATURE REVIEW

The simplest form of reinsurance is quota-share (QS) reinsurance, which involves a fully proportional sharing of risk. This type of reinsurance is widely used across almost all insurance sectors, particularly because of its conceptual clarity and administrative simplicity. In general, the primary insurer cedes a proportional share of the premiums to the reinsurer as well. Another common type of reinsurance contract is the excess of loss (XL) contract, which is frequently discussed in the literature because of its transparency. Under this arrangement, the reinsurer agrees to cover the portion of each claim that exceeds the retention limit (Albrecher et al., 2017).

An early pricing model for catastrophe reinsurance was introduced by Strickler (1960, as cited in Chao, 2022). In that study, the author assumed a constant and deterministic catastrophe intensity. Although this assumption yielded an elegant pricing formula, it prevented the use of a statistical framework for updating the model on the basis of newly observed data. To address the limitations of the models presented by Ekheden and Hossjer (2014) and Strickler (1960, as cited in Chao, 2022, p. 4473), Chao (2022) proposed a new pricing model that combined a compound Poisson process with the Peaks Over Threshold (POT) approach. Using the standard deviation premium principle, Chao (2022) derived an approximate price for a CAT XL contract through Monte Carlo simulation techniques. Subsequently, Leppisaari (2016) further refined the pricing framework developed by Ekheden and Hossjer (2014) by introducing a microsimulation approach for catastrophe reinsurance pricing. Xiao and Meng (2015) derived explicit pricing formulas for CAT XL contracts under three distinct retention scenarios, based on the assumption that the catastrophe reinsurance premium is determined according to the expectation principle.

The concept of layered pricing, based on different attachment points in excess-of-loss reinsurance contracts, was examined by Yue et al. (2016) through the application of Extreme Value Theory (EVT). In their empirical analysis, the authors successfully fitted the POT model to historical typhoon loss data from Zhejiang Province to estimate the pure premium associated with typhoon risk. Their methodology provided a clear framework for valuing layered reinsurance coverage through the explicit modeling of extreme loss events.

Li et al. (2016) applied Bayesian methods to analyze earthquake catastrophe risk and proposed four different mixture models to capture the complexity of earthquake loss distributions. Markov Chain Monte Carlo (MCMC)

techniques were used to estimate the unknown parameters and the critical threshold in the POT model. The results indicated that the estimated threshold, as well as the shape and scale parameters of the Generalized Pareto Distribution (GPD), varied considerably across models. In addition, Value at Risk (VaR) and Expected Shortfall (ES) were calculated for the proposed mixture models under different confidence levels.

By incorporating interest rate volatility, Chao (2021) priced a new type of multi-risk catastrophe reinsurance contract in which interest rate dynamics were modeled using the Cox–Ingersoll–Ross (CIR) process. It should be noted that Chao (2022), Ekheden and Hossjer (2014), and Leppisaari (2016) do not provide explicit pricing formulas for CAT XL contracts, which may limit pricing accuracy in practical applications.

Jaladri et al. (2021) estimated catastrophe reinsurance premiums in Indonesia by modeling event frequency with a Poisson distribution and claim severity using the DGPD for fatalities and the Beta–Binomial distribution for claim counts. The study then conducted numerical simulations to estimate the total annual claim amount. Finally, the risk premium was determined by applying the standard deviation premium principle to the simulated moments.

The influence of climate change risk on reinsurance pricing and available capacity was a central focus of Surya et al. (2023), who further advanced catastrophe reinsurance pricing by extending the traditional POT methodology within the EVT framework. Their principal contribution was the simultaneous incorporation of multiple CAT risk types, specifically house damage and fatalities. They developed a simulation-based structure for pricing double-risk CAT reinsurance under an XL treaty. By fitting the POT model to empirical earthquake loss data from Indonesia, the authors derived pricing structures consistent with the standard deviation premium principle for these more complex multi-peril contracts.

Extreme value theory

Catastrophic risks exhibit a defining characteristic: a low frequency of occurrence combined with the potential for extremely high losses, which directly corresponds to a heavy-tailed loss distribution. To analyze this behavior, EVT is primarily used to model the tail behavior and distributional properties of the relevant random variables. EVT provides two principal methods for characterizing extreme-value behavior based on observed data: the Block Maximum Method (BMM) and the POT method. The BMM involves dividing the data into blocks and then modeling only the maximum (or minimum) observation within each block. In contrast, the POT method first specifies a threshold and then models all observations exceeding that threshold. A major drawback of the BMM is that, by using only the maximum observation in each block, it discards a substantial amount of potentially relevant data that still lie in the extreme tail but do not represent the absolute maximum within their respective blocks. To make fuller use of the available data, this paper adopts the POT approach to fit earthquake catastrophe losses. This approach focuses on the asymptotic distribution of sample observations exceeding the selected high threshold.

Assume that $X_1, X_2, \ldots, X_n$ are i.i.d. random variables with the distribution function F. Let $Y = W - d$, where W= αX, so the distribution of Y is referred to as the conditional excess distribution function.

$$F_d(y) = \Pr(W - d \le y \mid W > d) = \frac{F(d+y) - F(d)}{1 - F(d)} \quad ; y \ge 0.$$

Based on the Balkema–de Haan–Pickands theorem (Klugman et al., 2019), the right tail of the excess-loss distribution belongs asymptotically to the class of generalized Pareto distributions. Therefore, for sufficiently large d>0, $F_d(y)$ can be well approximated by G, namely:

$$F_d(y) \approx G_{\varepsilon,\sigma} = \begin{cases} 1 - (1 + \varepsilon \frac{x}{\sigma})^{\frac{-1}{\varepsilon}} & \varepsilon > 0, \ x \ge 0 \\ 1 - \exp\left(\frac{-x}{\sigma}\right) & \varepsilon = 0, \ x \ge 0 \\ 1 - (1 + \varepsilon \frac{x}{\sigma})^{\frac{-1}{\varepsilon}} & \varepsilon < 0, \ 0 \le x \le \frac{-\sigma}{\varepsilon} \end{cases}$$

where ε and σ are the shape and scale parameters, respectively. Note that if $\varepsilon > \frac{1}{2}$, the variance of GPD does not exist. Therefore, we assume $\varepsilon \le \frac{1}{2}$ from this point onward to ensure the price of CAT QS-XL contract can be calculated using the standard deviation premium principle.

Pricing formulas of CAT QS-XL contracts

In this article, we consider the following assumptions:

$A_1 : X_i, i = 1, \ldots, n,$ are independent random variables representing the number of fatalities resulting from a particular event;

A_2 : Let $N(t)$ denote the number of catastrophic events occurring in the time interval $[0,t]$, where $t \ge 0$ is measured in years. The claim arrival process $\{N(t), t \ge 0\}$ is assumed to be a homogeneous Poisson process with constant intensity $\lambda > 0$. Accordingly, $N(0) = 0$, the increments of $N(t)$ are independent, and for $0 \le s < t$, $N(t) - N(s)$ represents the number of claims in the interval $(s,t]$. Consequently, for each $t \ge 0$, $N(t)$ follows a Poisson distribution with parameter λt and its expectation is given by $E[N(t)] = \lambda t$ (Ross, 2014).

A_3 : d denotes the cedant's retention;

A_4 : α is a proportionality factor satisfying $0<\alpha<1$.

The following definitions provide the general concept of the quota-share and excess-of-loss reinsurance treaties.

Definition 1. Under quota-share reinsurance treaty, denoted by QS, the reinsurer accepts a fixed percentage α of each claim. In the other word, the cedant's share is $(1-\alpha)X_i$ and the reinsurance's share is αX_i.

Definition 2. An excess-of-loss reinsurance treaty, denoted by XL, is a non-proportional reinsurance contract under which the reinsurer pays only for claims that enter the reinsured layer. Typically, this coverage is provided only up to a specified limit *L*,

$$X_i^{reinsurer} = \min\left(L,(X_i-d)_+\right)$$

where $(X_i-d)_+ = \max(0,X_i-d)$ (Albrecher et al., 2017).

In this paper, a combination of proportional reinsurance and excess-of-loss reinsurance is used for CAT reinsurance. This hybrid structure operates as follows: first, the quota-share contract is applied, meaning that the insurer remains responsible only for its contractual share of any claim arising from the risk. The insurer's share is determined according to the terms of the contract. Subsequently, the excess-of-loss contract is applied, meaning that the insurer pays only the amount exceeding a specified fixed threshold for any claim that occurs.

In a reinsurance framework where an insurance company seeks coverage for n independent risks, each defined as either a single policy or a homogeneous group of policies, the company may choose among three structural options: a pure quota-share treaty, an excess-of-loss treaty, or a combination of both. Under the hybrid form, the reinsurance operates in two sequential stages. First, the quota-share contract is applied, under which the reinsurer accepts a fixed proportion α (where $0<\alpha<1$) of each claim amount X_i as specified in the contract. Subsequently, the excess-of-loss contract applied, establishing a fixed upper bound d on the insurer's claim liability. Mathematically, for the i^{th} risk, the reinsurer's share becomes:

$$X_i^{reinsurer}(\alpha,d) = \min(L,\max(0,\alpha X_i-d)) = \min(L,(\alpha X_i-d)_+).$$

This layered mechanism provides a dual risk-sharing effect: the quota-share component ensures proportional participation across all claim levels, whereas the excess-of-loss component imposes a strict cap on the insurer's exposure to extreme losses. Consequently, this hybrid structure offers improved capital allocation efficiency and optimal premium calibration for catastrophe-prone portfolios.

Let N (t) to be a Poisson process with constant arrival rate λ and d denote the cedant's retention. X_i represents the number of fatalities resulting from a particular event, and X_i is independent of N. If the aggregate claim of the ceding company arising from a single catastrophic event exceeds the retention d, the excess W_i-d will be paid by the reinsurer. Note that $W_i=\alpha X_i$. Therefore, the reinsurer will pay the total claim amount

$$R=\sum_{i=1}^{N(t)} c\times min(L,(W_i-d)_+) \coloneqq \sum_{i=1}^{N(t)} C_i,$$

where c is the claim coefficient associated with death and $(W_i-d)_+ = max(0,W_i-d)$.

Using the standard deviation premium principle, the CAT QS-XL contract price P is given by

$$P=E(R)+\rho\sqrt{Var(R)}, \quad (1)$$

where ρ is a loading factor, typically $\rho\in[0.1,0.5]$ and E(R) and Var(R) denote the pure premium and variance, respectively, which can be calculated as follows:

$$E(R)=E(N)E(C_i)=\lambda E(C_i)$$
$$=\lambda c\left[\int_d^{d+L}(W_i-d)f(W_i)dW_i+L\int_d^{\infty}f(W_i)dW_i\right].$$

Using integration by parts, we obtain:

$$=\lambda c\left[(W_i-d)(-S(W_i))\Big|_d^{d+L}+\int_d^{d+L}S(W_i)dW_i+LS(D+L)\right]=$$
$$\lambda c\alpha\int_d^{d+L}(1+\varepsilon\frac{x_i}{\sigma})^{-\frac{1}{\varepsilon}}dX_i.$$

Using a change of variable $y=(1+\varepsilon\frac{x_i}{\sigma})^{-\frac{1}{\varepsilon}}$, the resulting integral is as follows:

$$E(R)=\frac{\lambda c\alpha\sigma}{\varepsilon-1}[(1+\varepsilon\frac{d+L}{\sigma})^{1-\frac{1}{\varepsilon}}-\left(1+\varepsilon\frac{d}{\sigma}\right)^{1-\frac{1}{\varepsilon}}].$$

Similarly, for the second moment, we have:

$$E(R^2)=2\lambda\alpha^2c^2S(d).\frac{\sigma_d^2}{\varepsilon}\left[\left(\frac{\vartheta_L^{2-\frac{1}{\varepsilon}}}{2\varepsilon-1}-\frac{\vartheta_L^{1-\frac{1}{\varepsilon}}}{\varepsilon-1}\right)+\frac{\varepsilon}{(2\varepsilon-1)(\varepsilon-1)}\right]$$

where $S(d)=(1+\varepsilon\frac{d}{\sigma})^{-\frac{1}{\varepsilon}}$, $\sigma_d=\sigma+\varepsilon d$, $\vartheta_L=1+\varepsilon\frac{L}{\sigma_d}$.

$$Var(R)=E(R^2)-(E(R))^2.$$

Let n be the sample size and let n_d be the number

of observations exceeding the threshold d. According to the historical simulation method, $S(d)=1-F(d)$ can be estimated by $\frac{n_d}{n}$. Finally, the standard deviation premium formula is as follows:

$$P=\frac{n_d}{n}\frac{c\alpha\lambda\sigma_d\left[1-\vartheta_L^{1-\frac{1}{\varepsilon}}\right]}{(1-\varepsilon)}+$$
$$\rho c\alpha\sigma_d\sqrt{\left[\frac{2n_d}{n\varepsilon}\left(\frac{\vartheta_L^{2-\frac{1}{\varepsilon}}-1}{2\varepsilon-1}-\frac{\vartheta_L^{1-\frac{1}{\varepsilon}}-1}{\varepsilon-1}\right)-\left(\frac{n_d}{n(1-\varepsilon)}.\left(1-\vartheta_L^{1-\frac{1}{\varepsilon}}\right)\right)^2\right]}. \quad (2)$$

A reinsurance company generally does not assume unbounded loss exposure; therefore, an upper limit for coverage must be specified. This limit, referred to as the exit point, corresponds to the ceiling of the reinsurance layer. A common actuarial choice for defining this threshold is the 99th or 99.5th percentile of the loss distribution.

The exit point is then determined using the Value at Risk (VaR) measure. Given the estimated parameters of the GPD, the VaR at a specified confidence level p is obtained through the quantile function, i.e., the inverse of the cumulative distribution function (CDF). This enables estimation of the number of fatalities associated with a probability level such as 99%. The formula for computing the p-th percentile of the loss distribution is given as follows:

$$N_{max}=\frac{\sigma}{\varepsilon}[(1-p)^{-\varepsilon}-1]\cdot$$

And

$$L=N_{max}-d\cdot \quad (3)$$

MAIN RESULTS

Building upon the theoretical foundations established in the previous section, this section applies the proposed QS–XL pricing framework, integrated with the POT model, to empirical data in order to evaluate its numerical consistency and practical applicability.

For the numerical implementation, data on earthquake events in Iran covering the period 1950–2023 are obtained from the https://www.worlddata.info/asia/iran/earthquakes.php database. The number of fatalities across the recorded events ranges from 0 to 50,000. As illustrated in Fig. 1, the empirical distribution of fatalities exhibits a pronounced right-skewed pattern.

Fig. 2, which displays annual earthquake fatalities (number of deaths) over time from 1950 to 2023, provides visual evidence consistent with the heavy-tailed nature of catastrophic risk in Iran. This observation supports the use of EVT and the POT method for modeling the data.

The time series is characterized by long periods of low or near-zero fatalities, punctuated by a small number of exceptionally high-severity events. This pattern is consistent with the behavior typically observed in heavy-tailed risk processes in actuarial science, where a large share of the aggregate risk is driven by a limited number of rare yet extreme events.

Low frequency: Most observations cluster near the horizontal axis, indicating that in the majority of years earthquake fatalities were minimal.

High severity: The overall risk is dominated by several pronounced spikes representing catastrophic events that determine the upper bound of potential financial exposure for insurers and reinsurers.

The data reveal several extreme events that must be considered in any catastrophe risk model.

The year 1990 represents the maximum observation in the series, corresponding to the Manjil–Rudbar earthquake. This event represents an extremely severe realization and is relevant for assessing the upper range of potential losses in solvency and capital requirement analyses.

The year 2003 corresponds to the second largest spike, associated with the Bam earthquake. The presence of this event, together with the 1990 peak, indicates that events producing fatalities above 30,000 are rare but non-negligible occurrences, reinforcing the importance of catastrophic reinsurance coverage.

Several additional significant peaks are observed and are particularly relevant for defining the region above the

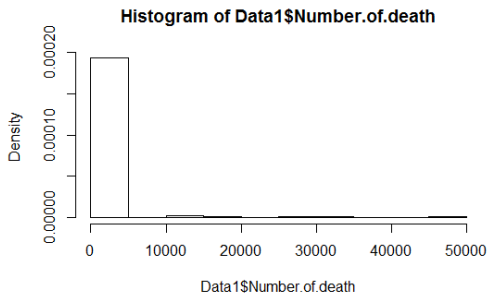


Fig. 1. Histogram of deaths

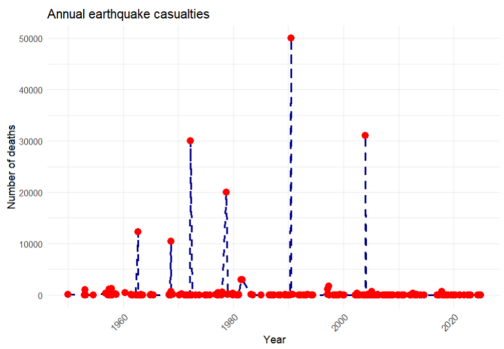


Fig. 2. Annual earthquake casualties

selected threshold (i.e., the "excesses" in the POT method).

a. 1978 peak: approximately 20,000 fatalities (likely associated with the Tabas earthquake).

b. 1962 peak: approximately 12,000 fatalities (likely associated with the Buyin-Zahra earthquake).

c. 1968 peak: approximately 10,000 fatalities.

These intermediate peaks indicate that the catastrophic layer of risk is not determined solely by the two largest observations but rather by a broader spectrum of high-impact events occurring over time.

The visual representation of these varying peak magnitudes highlights the importance of appropriate threshold selection in the GPD modeling framework.

If a low threshold is selected, the model would include a large number of relatively small exceedances, which may bias the estimated GPD parameters and lead to inaccurate estimation of the probability of extremely severe events (e.g., those approaching the 50,000-fatality range).

The descriptive statistics clearly indicate that the fatality data exhibit the main characteristics of catastrophic risk, including extreme asymmetry and leptokurtosis, which are consistent with a heavy-tailed distribution. A substantial proportion of the observations, 36%, recorded zero fatalities, which is expected for low-magnitude or low-impact seismic events. The disparity between the mean and the median is considerable, immediately indicating a highly skewed distribution. A median of one fatality suggests that the majority of events result in minimal loss of life, whereas the extreme values substantially increase the mean.

Skewness: A high positive value of 7.27 indicates that the distribution has a long right tail driven by rare, severe events.

Kurtosis: An exceptionally high kurtosis value of 57.54 confirms the leptokurtic nature of the distribution, meaning that the data contain substantially more extreme observations than would be expected under a normal distribution.

The ratio of the standard deviation (5,023.91) to the mean (883.64), yielding a coefficient of variation of 5.69, highlights the extraordinary volatility and unpredictability of the losses. Furthermore, the rapid escalation in loss severity is illustrated by the quantiles: whereas the 75th percentile is only 16.50 deaths, the 95th percentile rises sharply to 1,130.00 deaths, and the maximum observed value is 50,000. This behavior supports the application of EVT, specifically the POT method, as standard statistical methods are generally inadequate for modeling such a heavy-tailed distribution. The POT approach, using an appropriate high threshold, is therefore justified for modeling the excess losses.

The geographical distribution of earthquake events is important for understanding the underlying catastrophe risk exposure across Iran.

The spatial analysis of the dataset confirms the broad geographical vulnerability of the region. The data exhibit substantial geographical dispersion. Out of the total number of events, 160 unique regions were identified. This implies that approximately 82% of the recorded events occurred in a location that was unique within the dataset, or, equivalently, that a large majority of event locations appeared only once. This high degree of uniqueness suggests that the catastrophic risk is not confined to a single highly concentrated area but instead represents a nationwide, systemic risk.

Although the geographical dispersion is substantial, several regions appear more than once in the dataset. The term "Iran" is used for 7 events; in these cases, the exact location (city) is not specified and is recorded only as Iran. Kermanshah (6 events) is the most frequently cited specific region, followed by Hormozgan Province (4 events), and a group of other regions (Firuzabad, Gorgan, and Kerman, each with 3 events).

The wide geographical distribution of earthquake events supports the model's foundational assumption of a large, non-homogeneous risk portfolio, in which losses are driven by low-frequency, high-severity events that can materialize across a vast geographical area. This geographical heterogeneity is an important characteristic for pricing CAT reinsurance contracts, as it reduces the likelihood that minor, non-catastrophic events will exhibit strong geographical correlation.

Table 1. Descriptive statistics for earthquake fatalities (Number of deaths) 1950-2023

Min	Max	Mean	Median	SD	Kurtosis	Skewness
0	50000	883.6	1	5023.9	57.5	7.3

The hypocentral depth of an earthquake is a critical factor influencing the intensity of ground shaking and the resulting surface damage and casualties. The depth analysis reveals important characteristics regarding the origin of the seismic events.

A significant portion of the data, 38 events, has unspecified or missing depth information (represented by a blank entry). This missingness should be noted as a limitation in any depth-dependent vulnerability analysis, although it does not affect the fatality-based premium calculation.

For the recorded depths, there is a clear concentration of events in the shallow and mid-depth ranges. The most frequently recorded depth is 10 km (7.7% of all events), suggesting a high occurrence of crustal earthquakes. Depths of 2 km and 8 km are also highly represented.

The most frequently recorded depth levels indicate that a large proportion of the earthquakes in the dataset are shallow (typically defined as <70 km). Shallow earthquakes are generally associated with higher surface intensity and greater potential for catastrophic losses, which is consistent with the severe fatality counts observed in the broader dataset.

The prevalence of shallow-focus events underscores the high inherent hazard risk. Although depth is not directly used as a parameter in the POT model for loss severity, this observation is consistent with the need for robust reinsurance for catastrophe-prone portfolios, as a large number of recorded events originate from depths likely to cause severe damage. The depth distribution is highly heterogeneous, with 57 unique depth levels recorded, demonstrating the varied tectonic sources contributing to the catastrophe risk pool.

In summary, earthquake fatalities are strongly associated with extreme loss outcomes. Accordingly, the premium model is developed within the framework of extreme events in order to support CAT risk management through a QS–XL reinsurance contract.

The choice of threshold is highly important within the GPD modeling framework. Determining the optimal threshold for threshold models requires balancing the variance of the model against inherent systematic bias. A higher threshold restricts the amount of data available for inference, resulting in greater variance in the final estimates. Conversely, a lower threshold incorporates a larger volume of data into the analysis, thereby reducing the overall estimation variance (Chao, 2021).

There are several methods for selecting the threshold (retention level). One approach is to determine the insurer's retention share for each risk on the basis of existing regulatory requirements announced annually by the supervisory authority. However, due to the unavailability of the insurer's loss data, this method could not be applied in the present study.

A second and widely accepted approach is based on the mean excess plot, which is regarded as one of the most robust diagnostic tools in EVT for modeling excess losses in earthquake data using the GPD. The mean excess plot is constructed for different values of D, representing the number of fatalities. A threshold beyond which the plot exhibits approximately linear behavior is considered a suitable candidate.

As illustrated in Fig. 3, the mean excess plot appears to flatten at values of 3,000, 12,000, and 31,000 fatalities. Accordingly, a threshold of 32,000 is initially suggested. Nevertheless, this threshold is impractical for reinsurance applications, as the empirical data indicate that the number of fatalities exceeds this level in only one observation.

Another method for determining the threshold is based on the return period criterion. Under this approach, the insurer retains high-frequency, low-severity losses and transfers low-frequency, high-severity losses to the reinsurer. For example, the primary insurer may decide to retain claims associated with a return period shorter than T, while ceding losses with longer return periods to reinsurance coverage.

An alternative approach, proposed by Rydman (2018) and adopted in this study, is the percentage-based method, which determines the threshold as a fixed percentage of candidate threshold values. An analysis of the earthquake dataset indicates that approximately 73% of events are classified as minor.

As a preliminary step, the kurtosis of the observed data is examined to assess whether the distribution exhibits heavy-tailed behavior, with particular attention to whether the kurtosis exceeds three. The percentage-based method is then employed to determine an appropriate threshold value. Extreme observations are subsequently defined as those data points that exceed the selected threshold.

According to Rydman (2018), approximately 27% of the data are classified as extreme values. The steps of the percentage-based method (27%) are as follows:

Step 1. Sort the data from largest to smallest.

Step 2. Calculate the number of extreme observations using the following equation

K=27%×N,

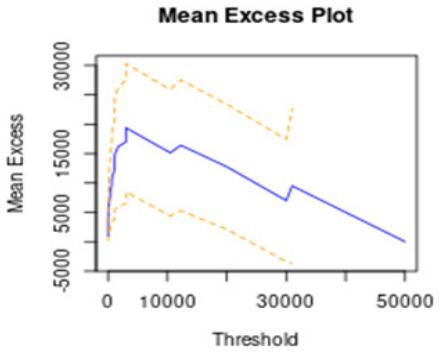


Fig. 3. Mean excess plot of deaths

Table 2. Premium calculated under the QS-XL hybrid reinsurance model

α	d	P (million IRR)
0.2	5	179,167,086
0.4	5	358,334,171
0.6	5	537,501,257
0.2	7	153,571,788
0.4	7	307,143,575
0.6	7	460,715,363
0.2	12	138,214,609
0.4	12	276,429,218
0.6	12	414,643,827

denote the number of extreme observations and the total sample size, respectively.

Step 3. Determine the threshold value using the following equation

U=K+1. (4)

Based on Equation (4), the number of extreme observations is 51. Therefore, the threshold value corresponds to the 52nd observation, which is equal to 12. Given the substantial gap between 12 and 50,000, assuming a reinsurance ceiling of 50,000 would result in a prohibitively high reinsurance premium. Such a structure would effectively leave the primary insurer with no meaningful retention of risk. Using Equation (3), a layer limit of L=11,466 is obtained. An examination of the dataset indicates that the number of fatalities exceeds this ceiling in only four earthquake events. As the majority of loss events fall within this layer, the corresponding reinsurance premium is expected to be substantial. Super-catastrophic events may therefore be treated as force majeure occurrences, which in practice may require government intervention.

To examine the impact of the threshold value on the reinsurance premium, several percentage-based threshold selection scenarios are considered. The corresponding results are reported in Table 2. As shown therein, the threshold value plays a crucial role in determining the reinsurance premium P.

Once an appropriate threshold has been identified, the parameters of the Generalized Pareto Distribution are estimated using the Maximum Likelihood Estimation (MLE) method. The estimated shape and scale parameters are 0.4823 and 2715.93, respectively.

The Kolmogorov–Smirnov test is employed to evaluate the following hypothesis:

H_0= The data follow a Generalized Pareto distribution.

The p-value of the test is 0.82032. Accordingly, the null hypothesis cannot be rejected at the 5% significance level.

Given that 187 hazardous events were observed over a 73-year period, the event frequency is modeled using a Poisson distribution with mean $\lambda = 2.56\left(=\frac{187}{73}\right)$. The parameter values are set as follows: $\rho = 0.3$, c=10,000,000 IRR, retention levels (number of fatalities) d=5, 7, 12 and quota-share factor $\alpha = 0.20, 0.40, 0.60$. The resulting CAT reinsurance premiums are reported in Table 2.

Table 2 indicates that, as the retention level d increases, the reinsurance premium decreases, reflecting the larger proportion of losses retained by the primary insurer and the corresponding reduction in the size of the reinsurance layer. Conversely, the premium changes with respect to the quota-share coefficient α. For a fixed value of α, the following relationship is observed:

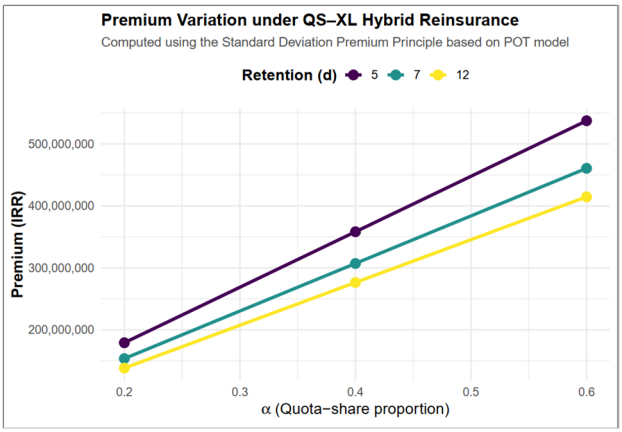


Fig. 4. Premium variation under the QS-XL hybrid reinsurance model

$$premium_{d=5} > premium_{d=7} > premium_{d=12}$$

Holding the retention level d constant, the results further show that:

$$premium_{\alpha=0.2} > premium_{\alpha=0.4} > premium_{\alpha=0.6}$$

This behavior indicates that changes in the reinsurer's share of the loss directly affect the reinsurance premium. Based on these results, the ceding company and the reinsurer may jointly select the threshold level that is most appropriate for determining the reinsurance premium.

Fig. 4 illustrates the evolution of the reinsurance premium under varying values of the QS coefficient α and different retention levels d. The curves show how the premium responds to changes in α across alternative values of d.

CONCLUSIONS

The distinctly heavy-tailed nature of catastrophe loss data substantially constrains reinsurers' willingness to underwrite direct catastrophe layers. To promote underwriting participation and ensure actuarially sound pricing, the premium calculation framework for such contracts adopts the standard deviation premium principle rather than the traditional net premium principle, as it explicitly accounts for the heightened uncertainty associated with extreme loss events.

In this study, the POT model is employed to fit empirical earthquake loss data, enabling an accurate characterization of tail behavior. Based on the fitted model, the parameters of the Generalized Pareto Distribution (GPD) are estimated using the Maximum Likelihood Estimation (MLE) method. Reinsurance premiums are then computed within a hybrid QS–XL structure across various combinations of quota-share coefficients α and retention levels d.

The empirical results demonstrate that reinsurance premiums are inversely related to the retention level d. Specifically, higher retention by the primary insurer reduces the size of the reinsurance layer, leading to lower premiums. Conversely, premiums increased linearly vary with respect to the quota-share coefficient α, reflecting changes in the reinsurer's share of losses. These monotonic relationships confirm the strong sensitivity of catastrophe reinsurance pricing to contractual design parameters.

Furthermore, a comprehensive sensitivity analysis reveals that premium estimates are highly responsive to threshold selection within the POT framework. Higher threshold percentiles consistently generate higher reinsurance premiums, highlighting the trade-off between underwriting capacity and loss severity in catastrophe risk transfer. Consequently, the choice of an appropriate threshold emerges as a critical negotiation parameter between ceding insurers and reinsurers, ensuring that premiums remain both statistically coherent and commercially viable.

Future research may extend the proposed framework by incorporating clustered extreme events or serial dependence structures, which may produce multiple large losses within a single portfolio year, thereby further enhancing the robustness and realism of QS–XL catastrophe risk pricing models.

CONTRIBUTION OF EACH AUTHOR

First author: Writing – original draft, Data gathering, Methodology, Data analysis, Conceptualization;

Second author: Writing – review & editing, Methodology, Conceptualization.

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CONFLICT OF INTEREST

The author(s) declare that there is no conflict of interest regarding the publication of this research. Furthermore, ethical considerations—including plagiarism, informed consent, misconduct, fabrication and/or falsification of data, duplicate publication, and the journal's policy on the use of artificial intelligence—have been fully observed by the authors.

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