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Pricing Catastrophic Risk under a Hybrid Reinsurance Treaty: Empirical Evidence from a Peaks-Over-Threshold Approach

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ABSTRACT

BACKGROUND AND OBJECTIVES: Catastrophic risk refers to the exposure of a large population or portfolio to extreme, low-frequency but high-severity events that can simultaneously lead to significant financial, human, and infrastructural losses. Such events extend beyond the ordinary volatility captured by conventional risk models and often challenge both the solvency and underwriting capacity of insurance and reinsurance markets. Examples include large-scale earthquakes, floods, droughts, and terrorism, all of which can severely disrupt national economies and require external financial support for recovery.

In the context of Iran—recognized as one of the most disaster-prone countries globally, particularly with respect to seismic activity—the management of catastrophe risk is not only an actuarial necessity but also a public policy imperative. Over the past three decades, the country has recorded more than 80,000 fatalities due to earthquakes alone, highlighting the urgent need for robust catastrophe risk modeling and scientifically grounded reinsurance pricing mechanisms. Traditional actuarial models that rely on limited historical data or frequency-severity distributions often fail to capture the heavy-tailed nature of catastrophic losses. This limitation motivates the adoption of advanced statistical and probabilistic techniques, such as Extreme Value Theory (EVT), which offer more accurate characterization of rare but devastating events.

The primary objective of this research is to develop and empirically validate a framework for pricing catastrophe (CAT) reinsurance using a hybrid reinsurance treaty that combines the advantages of Quota-Share (QS) and Excess-of-Loss (XL) arrangements. This hybrid structure allows flexible sharing of both frequency and severity risks across different loss layers, improving the efficiency and stability of catastrophe coverage.

METHODS: To quantify the tail behavior of catastrophic losses, earthquake fatality data from 1950 to 2023 in Iran are analyzed using the Peaks-Over-Threshold (POT) approach within the EVT framework. This method focuses on the exceedances above a selected high threshold, enabling precise modeling of extreme observations while avoiding bias from moderate events. The Generalized Pareto Distribution (GPD), a canonical model under POT, is employed to describe the distribution of excess fatalities. Its parameters are estimated via Maximum Likelihood Estimation (MLE), which ensures statistical efficiency and consistency under large samples.

Using the fitted GPD, the tail probabilities and expected loss beyond the threshold are integrated into a reinsurance pricing model based on the Standard Deviation Premium Principle. This principle, which incorporates both the expected value and the dispersion of losses, provides a theoretically sound measure for risk loading and premium adjustment. The hybrid QS–XL structure is mathematically formulated to capture the interplay between retention level (d) and quota-share coefficient (α), offering a unified framework to compute the reinsurer's expected payout and corresponding premium.

Furthermore, sensitivity analyses are performed across multiple retention levels and quota-share ratios to assess how changes in contractual parameters affect the premium dynamics. The model is calibrated under realistic assumptions for Iranian earthquake data, reflecting both empirical severity patterns and practical treaty configurations used by reinsurance companies operating in high-risk regions.

FINDINGS: Empirical analysis reveals a clear inverse relationship between the reinsurance premium and the retention level d. As the primary insurer retains a greater share of the loss, the reinsurance layer becomes narrower, resulting in a lower reinsurance premium. Conversely, the premium demonstrates a positive and approximately linear dependence on the QS coefficient (α)—that is, as the reinsurer's proportional participation increases, the overall price of coverage escalates.

The pattern of results confirms that reinsurance pricing under catastrophe conditions is highly sensitive to the structural parameters of the contract. For fixed quota-share proportions, higher retention levels considerably reduce premium costs; for fixed retention levels, increasing the reinsurer's share proportionally raises premiums due to higher expected indemnity. This empirical evidence highlights the trade-off between risk sharing and premium affordability, a core consideration for actuaries and policy designers in disaster-risk financing.

Moreover, the POT-based tail modeling allows capturing the extreme percentiles of the loss distribution, thereby providing more robust and transparent estimates of potential catastrophic payouts. Compared to conventional methods that might underestimate tail risk, the EVT-based approach ensures actuarial fairness and enhances resilience against large, unpredictable claims. The hybrid QS–XL model, supported by this tail-risk quantification, offers a balanced mechanism that suits both reinsurers and primary insurers seeking efficient coverage in highly vulnerable settings.

CONCLUSION: This study demonstrates that integrating Extreme Value Theory with hybrid reinsurance structures represents a powerful actuarial approach for pricing catastrophic risk. The proposed QS–XL framework effectively combines proportional and non-proportional elements, delivering analytical tractability and operational flexibility. By grounding the premium design in statistical inference and long-term empirical data, this approach strengthens the financial stability of both insurers and reinsurers operating in disaster-exposed regions.

In the case of Iran, where earthquake losses pose systemic threats to economic resilience, adopting this hybrid model could significantly improve capital adequacy, promote fair pricing of catastrophe coverage, and facilitate sustainable disaster-risk management. Beyond its local applicability, the model provides a replicable methodology for other emerging markets facing similar challenges in estimating and mitigating catastrophic risks.

Ultimately, the fusion of EVT-based tail estimation with hybrid treaty design bridges the gap between statistical rigor and real-world actuarial practice, paving the way for more resilient and data-driven catastrophe insurance frameworks worldwide.

1. Introduction

A catastrophic risk is fundamentally characterized by the simultaneous exposure of a large population to the potential for significant financial or physical loss stemming from a single, high-severity peril. Such events often manifest as natural calamities, including earthquakes, floods, or droughts, or as human-induced hazards such as large-scale terrorism, all leading to extensive loss of life and widespread infrastructure destruction.

Iran is a disaster-prone country. According to the global risk report, Iran ranks among the countries with the highest vulnerability worldwide, where natural disasters have caused significant financial, social and physical losses. During the past 30 years, more than 80,000 people have died directly as a result of natural disasters in Iran. Most of Iran's provinces are classified as arid and semi-arid regions. Throughout the history of Iran, there have been many natural disasters such as extreme floods, extreme temperatures and droughts. Additionally, the country is highly prone to earthquakes and has suffered many devastating seismic events (Seddighi & Seddighi, 2020).

Extreme events, by definition within the insurance context, are characterized by an exceptionally low probability of occurrence coupled with the potential for massive or devastating financial impact on the insurance industry. When such low-frequency, high-severity perils materialize, the resultant losses often exceed the capacity of a single insurer to absorb them independently. Consequently, the occurrence of these events can have catastrophic financial ramifications for individual underwriting entities.

Considering the potential for catastrophic events in Iran, effective management and control of catastrophic risk is important for insurance companies. Catastrophic (CAT) losses impose significant financial strain on primary insurers, potentially leading to insolvency. Consequently, transferring CAT risk to reinsurers is a vital risk management strategy for ceding companies. This raises a fundamental and critical question: what is the appropriate price of the CAT contract?

The purpose of this study is to develop the CAT reinsurance premium conducted by Chao (2021-a) into the QS-XL CAT reinsurance premium. The number of people who died is considered as a risk factor for calculating premium. The number of deaths is approximated using the POT model. The choice of this model is due to the high risk of CAT event. The standard deviation principle is applied to calculate the insurance premium. The rest of this article is organized as follows. Section 2 provides a brief overview of EVT. In Section 3, we derive the catastrophe reinsurance pricing formulas of CAT QS-XL contracts. Section 4 illustrates a practical application of our findings on a real dataset. Finally, conclusion and suggestions have been given in Section 5.

2. Theoretical Background and Literature Review

The simplest form of reinsurance is quota-share (QS) reinsurance, which is a fully proportional sharing of the risk. This form of reinsurance is popular in almost all insurance sectors, particularly due to its conceptual clarity and administrative simplicity. Generally, the primary insurer cedes to the reinsurer a proportionate share of the premiums as well. Another common type of reinsurance contract is the excess loss contract which is often used by researchers due to its transparency. Under this contract, the reinsurer agrees to cover the portion of each claim that exceeds the retention limit (Albrecher et al., 2017).

The early pricing model for catastrophe reinsurance was introduced by Strickler in 1960. In this work, the author assumed a constant, deterministic rate of catastrophes. While this assumption yielded an elegant pricing formula, it precluded any statistical approach for updating the model based on new incoming data. To address the drawbacks of the models presented in Strickler (1960), Ekheden & Hossjer (2014) proposed a novel pricing model by combining the compound Poisson process with the Peaks-over-Threshold (POT) model. Utilizing the standard deviation premium principle, they derived an approximate price for a Cat XL contract using Monte Carlo simulation techniques. Subsequently, Leppisaari (2016) further refined the pricing model established in Ekheden & Hossjer (2014) by introducing a micro-simulation approach for catastrophe reinsurance pricing. Xiao & Meng (2013) derived explicit pricing formulas for Cat XL contracts across three distinct retention scenarios, based on the assumption that the catastrophe reinsurance premium is determined using the expectation principle.

The concept of layered pricing, utilizing different attachment points for excess-of-loss reinsurance policies, was explored by Yue et al. (2016) through the lens of extreme value theory. In their empirical analysis, they successfully fitted the Peaks over Threshold model to historical typhoon loss data from Zhejiang Province to accurately determine the pure premium associated with typhoon risk. Their methodology demonstrated a clear path for valuing layered reinsurance coverage by explicitly modeling extreme loss events.

Li et al. (2016) employed Bayesian approaches to analyze earthquake catastrophic risk, proposing four distinct mixture models to effectively capture the complexity of earthquake loss distributions. They utilized Markov Chain Monte Carlo (MCMC) methods as the computational engine for estimating unknown parameters and the critical POT threshold within these models. Results showed that the estimation of the threshold and the shape and scale of Generalized Pareto Distribution are quite different. Value-at-risk and expected shortfall for the proposed mixture models was calculated under different confidence levels.

Considering the impact of interest rate volatility, Chao (2021-b) priced a new type of multi-risk catastrophe reinsurance contract where interest rate dynamics were described by the Cox-Ingersoll-Ross (CIR) model. It should be noted that Chao (2021-a), Ekheden & Hossjer (2014) and Leppisaari (2016) cannot provide an accurate price for a Cat XL contract due to the lack of explicit pricing formulas.

Jaladari (2021) calculated catastrophic reinsurance premiums in Indonesia by modeling event frequency via a Poisson distribution and claim severity using the DGPD for fatalities and Beta-Binomial for claim counts. The study then performed numerical simulations to estimate the total claim amount over one year. Finally, the required risk premium was determined by applying the standard deviation premium principle to the simulated moments.

The influence of climate change risk on reinsurance pricing and available capacity was a key focus for Surya et al. (2021) significantly advanced catastrophe reinsurance pricing by extending the traditional POT methodology within the framework of Extreme Value Theory (EVT). Their key contribution was the incorporation of multiple CAT risk types simultaneously, specifically modeling both house damage and deaths events. They developed a simulation structure to determine premiums for double risk CAT reinsurance under an XL treaty. By fitting the POT model to empirical Indonesian earthquake loss data, the authors successfully derived pricing structures consistent with the standard deviation premium principle for these complex, multi-peril contracts.

3. Extreme value theory

Catastrophic risks exhibit a defining characteristic: a low frequency of occurrence juxtaposed with the potential for severely high losses, which directly translates to a heavy-tailed distribution in loss data. To analyze this behavior, Extreme Value Theory is primarily employed to model the tail behavior and distribution characteristics of relevant random variables. EVT offers two principal methodologies for characterizing extreme value behavior based on observed data: the Block Maximum Method (BMM) and the POT method. The BMM involves dividing the data into blocks and subsequently modeling only the maximum (or minimum) value within each block. Conversely, the POT method first establishes a threshold, and then models all observations that exceed this level. A significant drawback of BMM is that by exclusively utilizing the maximum per block, it discards vast amounts of potentially relevant data that still reside in the extreme tail but do not constitute the absolute maximum of their respective blocks. To ensure comprehensive utilization of available data, this paper adopts the POT model, for fitting earthquake catastrophe losses. This approach focuses on the asymptotic distribution of sample observations exceeding the selected high threshold.

Assume that $X_1, X_2, \dots, X_n$ are i.i.d. random variables with the distribution function F. Let $Y = W - d$, where W= αX, so the distribution of Y is called the conditional excess distribution function.

$$F_d(y) = \Pr(W - d \le y | W > d) = \frac{F(d+y) - F(d)}{1 - F(d)} \quad ; y \ge 0.$$

Based on Balkema-de Haan- Pickands Theorem (Klugman et al., 2019), the right-hand tail of the distribution of the excess coverage is shape to exactly one of the three generalized Pareto distributions. So, for large d>0, $F_d(y)$can be well approximated by G, namely

$$F_d(y) \approx G_{\varepsilon,\sigma} = \begin{cases} 1 - (1 + \varepsilon \frac{x}{\sigma})^{\frac{-1}{\varepsilon}} & \varepsilon > 0,\ x \ge 0 \\ 1 - \exp\left(\frac{-x}{\sigma}\right) & \varepsilon = 0,\ x \ge 0 \\ 1 - (1 + \varepsilon \frac{x}{\sigma})^{\frac{-1}{\varepsilon}} & \varepsilon < 0,\ 0 \le x \le \frac{-\sigma}{\varepsilon} \end{cases}$$

where ε and σ are the shape parameter and the scale parameter respectively. Note that if $\varepsilon > \frac{1}{2}$, the variance of GPD does not exist. We assume $\varepsilon \leq \frac{1}{2}$ from now on to ensure the price of CAT QS-XL contract can be calculated by the standard deviation premium principle.

4. Pricing formulas of CAT QS-XL contracts

In this article, we consider the following assumptions:

A_1: $X_i, i = 1, \dots, n$, are independent random variables representing the number of deaths resulting from a particular event;

A_2: Let $N(t)$ denote the number of catastrophic events occurring in the time interval $[0, t]$, where $t \geq 0$ is measured in years. The claim arrival process $\{N(t), t \geq 0\}$ is assumed to be a homogeneous Poisson process with constant intensity $\lambda > 0$. Accordingly, $N(0) = 0$, the increments of $N(t)$ are independent, and for $0 \leq s < t$, $N(t) - N(s)$ represents the number of claims in the interval $(s, t]$. Consequently, for each $t \geq 0$, $N(t)$ follows a Poisson distribution with parameter λt and its expectation is given by $E[N(t)] = \lambda t$ (Ross, 2014).

A_3: d denotes the cedant's retention;

A_4: α is a proportionality factor that satisfies $0 < \alpha < 1$.

The following definitions provide the general concept of the Quota-share and excess-of-loss reinsurance treaty.

Definition 1. Under Quota-share reinsurance treaty, say (QS), the reinsurer accepts a fixed percentage α of each claims. In the other word, the cedant part is $(1-\alpha)X_i$ and the reinsurance part is αX_i.

Definition 2. The excess-of-loss reinsurance treaty, say XL, is a non-proportional reinsurance contract in which the reinsurer only has to pay for claims which touch the reinsured layer. Typically, this will only be agreed upon up to a certain limit L,

$$X_i^{reinsurer} = \min\left(L, (X_i - d)_+\right)$$

Where $(X_i - d)_+ = \max(0, X_i - d)$. (Albrecher et al., 2017)

In this paper, a combination of a proportional reinsurance and excess-of-loss reinsurance contract is used for CAT reinsurance. The way this combination operates is as follows: first, the quota share contract is applied which means the insurer remains responsible for no more than its share of any claims that may occur for the risk. The insurer's share is determined based on the contract. Afterwards, the excess-of-loss contract is applied, which means the insurer shall pay only for the amount exceeding a certain fixed threshold of any claims that takes place.

In a reinsurance framework where an insurance company seeks coverage for n independent risks—each defined as either a single policy or a homogeneous group of policies—the company may choose among three structural options: a pure quota-share treaty, an excess-of-loss treaty, or a combination of both. Under the hybrid form, the reinsurance operates in two sequential stages. First, the quota-share contract applies, under which the insurer retains a fixed proportion α (where $0 < \alpha < 1$) of each claim amount X_i as specified in the contract. Subsequently, the excess-of-loss contract applies, establishing a fixed upper bound d on the insurer's claim liability. Mathematically, for the i^{th} risk, the reinsurer's share becomes:

$$\begin{aligned} X_i^{reinsurer}(\alpha, d) &= \min\left(L, \max(0, \alpha X_i - d)\right) \\ &= \min\left(L, (\alpha X_i - d)_+\right). \end{aligned}$$

This layered mechanism provides a dual risk-sharing effect: the quota-share component ensures proportional participation across all claim levels, while the excess-of-loss component imposes a strict cap on the insurer's exposure to extreme losses. Consequently, this hybrid structure offers improved capital allocation efficiency and optimal premium calibration for catastrophe-prone portfolios.

Consider N (t) to be a Poisson process with a constant arrival rate λ and d to be the cedant's retention. X_i number of deaths resulting from a particular event, and X_i is independent of N. If the aggregate claim of the ceding company of a single catastrophic event exceeds the retention d, the excess $W_i - d$ will be paid by the reinsurer. Note that $W_i = \alpha X_i$. Therefore, the reinsurer will pay the total amount of claims

$$R = \sum_{i=1}^{N(t)} c \times min(L, (W_i - d)_+) \coloneqq \sum_{i=1}^{N(t)} C_i,$$

where c is the claim coefficient of the death and $(W_i - d)_+ = max(0, W_i - d)$.

Using the standard deviation premium principle, the CAT QS-XL contract price P is given by

$$P = E(R) + \rho\sqrt{Var(R)}, \qquad (1)$$

where ρ is a loading factor with typically $\rho \in [0.1,0.5]$ and E(R) and Var(R) denote the pure premium and variance which can be calculated as follows:

$$E(R) = E(N)E(C_i) = \lambda E(C_i)$$

$$= \lambda c \left[\int_d^{d+L} (W_i - d)\, f(W_i) dW_i \right.$$
$$\left. + L \int_d^{\infty} f(W_i) dW_i \right]$$

Using integration by parts, we have:

$$= \lambda c \left[(W_i - d)\big(-S(W_i)\big)|_d^{d+L} + \int_d^{d+L} S(W_i)\, dW_i \right.$$
$$\left. + LS(D + L)\right]$$
$$= \lambda c\alpha \int_d^{d+L} (1 + \varepsilon \frac{x_i}{\sigma})^{-\frac{1}{\varepsilon}} dX_i$$

Using a change of variable $y = (1 + \varepsilon\frac{x_i}{\sigma})^{-\frac{1}{\varepsilon}}$, the resulting integral is as follows:

$$E(R) = \frac{\lambda c\alpha\sigma}{\varepsilon - 1}[(1 + \varepsilon\frac{d + L}{\sigma})^{1-\frac{1}{\varepsilon}} - (1 + \varepsilon\frac{d}{\sigma})^{1-\frac{1}{\varepsilon}}]$$

Similarly for the second moment we have:

$$E(R^2) = 2\lambda\alpha^2 c^2 S(d).\frac{\sigma_d^2}{\varepsilon}[\left(\frac{\vartheta_L^{2-\frac{1}{\varepsilon}}}{2\varepsilon - 1} - \frac{\vartheta_L^{1-\frac{1}{\varepsilon}}}{\varepsilon - 1}\right)$$
$$+ \frac{\varepsilon}{(2\varepsilon - 1)(\varepsilon - 1)}]$$

Where $S(d) = (1 + \varepsilon\frac{d}{\sigma})^{-\frac{1}{\varepsilon}}, \sigma_d = \sigma + \varepsilon d, \vartheta_L = 1 + \varepsilon\frac{L}{\sigma_d}$.

$$Var(R) = E(R^2) - (E(R))^2$$

Let n be the sample size and n_d be the number of the sample exceeding the threshold d. According to the historical simulation method, $S(d) = 1 - F(d)$ can be estimated by $\frac{n_d}{n}$. Finally, the standard deviation premium formula is as follows:

$$P$$
$$= \frac{n_d}{n}\frac{c\alpha\lambda\sigma_d[1 - \vartheta_L^{1-\frac{1}{\varepsilon}}]}{(1 - \varepsilon)}$$
$$+ \rho c\alpha\sigma_d\sqrt{[\frac{2n_d}{n\varepsilon}\left(\frac{\vartheta_L^{2-\frac{1}{\varepsilon}} - 1}{2\varepsilon - 1} - \frac{\vartheta_L^{1-\frac{1}{\varepsilon}} - 1}{\varepsilon - 1}\right) - (\frac{n_d}{n(1 - \varepsilon)}.(1}$$
$$- \vartheta_L^{1-\frac{1}{\varepsilon}}))^2] \quad (2)$$

A reinsurance company generally does not assume unbounded loss exposure; therefore, an upper limit to coverage must be specified. This limit, referred to as the exit point, corresponds to the ceiling of the reinsurance layer. A common actuarial choice for defining this threshold is the 99th or 99.5th percentile of the loss distribution.

The exit point is subsequently determined using the Value at Risk (VaR) measure. Given the estimated parameters of the GPD, the VaR at a specified confidence level

p is obtained through the quantile function, i.e., the inverse of the cumulative distribution function (CDF). This allows the estimation of the number of fatalities associated with a probability level of, for example, 99%. The formula for computing the p-th percentile of the loss distribution is given as follows:

$$N_{max} = \frac{\sigma}{\varepsilon}[(1 - p)^{-\varepsilon} - 1]$$

And

$$L = N_{max} - d \qquad (3)$$

5. Main results

Building upon the theoretical foundations established in the previous section, this section applies the proposed QS–XL pricing framework, integrated with the POT model, to empirical data in order to assess its numerical consistency and practical performance.

For the numerical implementation, data on earthquake events in Iran covering the period 1950–2023 are obtained from the https://www.worlddata.info/asia/iran/earthquakes.php database. The number of fatalities ranges from 0 to 50,000 across the observed events. As illustrated in Figure 1, the empirical distribution of fatalities exhibits a pronounced right-skewed pattern.

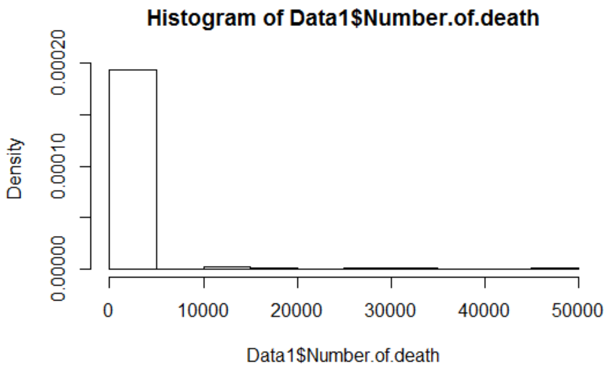

Figure 1. Histogram of deaths

Figure 2, displaying annual earthquake fatalities (Number of deaths) over time (from 1950 to 2023), offers a critical visual confirmation of the underlying heavy-tailed nature of catastrophic risk in Iran. This visual evidence strongly supports the choice of EVT and the POT method for modeling the data.

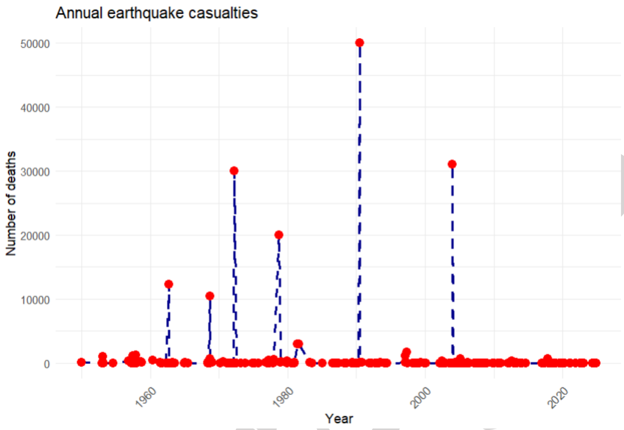

Figure 2. Annual earthquake casualties

The time series is characterized by long periods of low-to-zero casualties, abruptly punctuated by a few exceptionally high-severity events. This pattern is the defining feature of heavy-tailed distributions in actuarial science, where the majority of total risk is concentrated in a handful of rare occurrences.

Low Frequency: Most data points cluster near the x-axis, indicating that in the vast majority of years, earthquake fatalities were minimal.

High Severity: The total risk is dominated by several massive spikes, which represent the catastrophic events that define the maximum financial exposure for insurers and reinsurers.

The data reveals distinct extreme events that must be accounted for in any risk model:

Year 1990, Represents the absolute maximum observation in the series. This corresponds to the Manjil-Rudbar earthquake, an event of unparalleled severity, which dictates the upper bound of potential losses for solvency and capital requirements.

Year 2003, the second largest spike, corresponding to the Bam earthquake. The inclusion of this event, along with the 1990 peak, demonstrates that events causing fatalities above 30,000 are rare but recurring phenomena, solidifying the need for robust catastrophic reinsurance.

Several other significant peaks exist, which are crucial for defining the region immediately above the threshold (the "Excesses" in the POT method):

a. 1978 Peak: Approximately 20,000 fatalities (likely the Tabas earthquake).

b. 1962 Peak: Approximately 12,000 fatalities (likely the Buyin-Zahra earthquake).

c. 1968 Peak: Approximately 10,000 fatalities.

These intermediate peaks confirm that the catastrophic layer of risk is not composed of only the top two events, but rather a spectrum of high-impact events occurring over time.

The visual representation of these varying peak heights directly underscores the importance of the threshold selection in the GPD modeling framework: If a low threshold is chosen, the model would incorporate a large volume of small excesses, potentially biasing the GPD parameters and misestimating the probability of the truly massive events (50,000-death range).

Table 1. Descriptive statistics for earthquake fatalities (Number of deaths) 1950-2023

Min	Max	Mean	Median	SD	Kurtosis	Skewness
0	50000	883.6	1	5023.9	57.5	7.3

The descriptive statistics clearly indicate that the fatality data is highly characteristic of catastrophic risk, exhibiting extreme asymmetry and leptokurtosis, which confirms its heavy-tailed nature. A significant portion of the observations, 36%, recorded zero fatalities, which is expected for low-magnitude or low-impact seismic events. The disparity between the mean and the median is profound, immediately signaling a highly skewed distribution. The median of one fatality suggests that the majority of events result in minimal loss of life, yet the massive extreme values dramatically pull the mean upward.

Skewness: A high positive value of 7.27 indicates that the distribution possesses a long, heavy right tail, driven by rare, severe events.

Kurtosis: An exceptionally high kurtosis value of 57.54 confirms the leptokurtic nature of the distribution, meaning the data contains far more extreme outliers than would be present in a normal distribution.

The ratio of the Standard Deviation (5,023.91) to the mean (883.64), resulting in a coefficient of variation of 5.69, highlights the extraordinary volatility and unpredictability of the losses. Furthermore, the quick escalation in loss severity is illustrated by the quantiles: while the 75th percentile is only 16.50 deaths, the 95th percentile rises sharply to 1,130.00 deaths, and the maximum observed value is 50,000. This behavior necessitates the application of EVT, specifically the POT method, as the standard statistical methods are inadequate for modeling such a heavy-tailed distribution. The POT approach, using an appropriate high threshold, is therefore justified for modeling the excess losses.

The geographical spread of the earthquake events is crucial for understanding the underlying catastrophe risk exposure across Iran.

The spatial analysis of the dataset confirms the broad geographical vulnerability of the region. The data exhibits extremely high geographical dispersion. Out of total events, 160 unique regions were identified. This means that approximately 82% of the recorded events occurred in a location that was unique within the dataset, or that a large majority of event locations appeared only once. This high degree of uniqueness strongly suggests that the catastrophic risk is not confined to a single, hyper-concentrated area but is instead a nationwide, systemic risk.

Although the dispersion is very high, a few more regions are mentioned: the term "Iran" is used for 7 events, in these cases, the exact location (city) is not specified in the data and is only recorded as Iran. Kermanshah (6 events) is the most frequently cited specific region, followed by Hormozgan Province (4 events), and a group of other regions (Firuzabad, Gorgan, and Kerman, each with 3 events).

The wide distribution of earthquake events supports the model's foundational assumption of a large, non-homogenous risk portfolio, where losses are driven by low-frequency, high-severity events that can materialize across a vast geographical area. This geographic heterogeneity is a crucial characteristic for pricing CAT reinsurance contracts, as it reduces the likelihood that minor, non-catastrophic events will exhibit strong geographical correlation.

The hypo central depth of an earthquake is a critical factor influencing the intensity of ground shaking and the resulting surface damage and casualties. The depth analysis reveals crucial characteristics regarding the origin of the seismic events:

A significant portion of the data, 38 events, has unspecified or missing depth information (represented by a blank level). This missingness should be noted as a limitation in any depth-dependent vulnerability analysis, though it does not impact the fatality-based premium calculation.

For the recorded depths, there is a clear concentration of events in the shallow and mid-depth ranges: The most frequently recorded depth is 10 km (7.7% of all events), suggesting a high occurrence of crustal earthquakes. Depths of 2 km and 8 km are also highly represented.

The top recorded depth levels indicate that a large proportion of the earthquakes in the dataset are shallow (defined as typically <70km). Shallow earthquakes are generally associated with higher surface intensity and greater potential for catastrophic losses, which aligns with the severe fatality counts observed in the broader dataset.

The prevalence of shallow focus events underscores the high inherent hazard risk. While depth is not directly used as a parameter in the POT model for loss severity, the observation is consistent with the need for robust reinsurance for cat-prone portfolios, as a high number of recorded events originate from depths likely to cause maximum damage. The depth distribution is highly heterogeneous, with 57 unique depth levels recorded, demonstrating the varied tectonic sources contributing to the catastrophe risk pool.

In summary, deaths caused by earthquake are highly associated with extreme death. We established the premium model in the context of extreme events to conduct CAT risk management through QS-XL reinsurance contract.

The choice of the threshold holds undeniable significance within the GPD modeling framework. Determining the optimal threshold for threshold models necessitates balancing the model's variance against inherent systematic bias. A higher threshold selection restricts the dataset available for inference, resulting in increased variance in the final estimation. Conversely, opting for a lower threshold incorporates a larger volume of data into the analysis, thereby leading to a reduction in the overall estimation variance, (Chao, 2021-b).

There are several methods for selecting the threshold (retention level). One approach is to determine the insurer's retention share for each risk based on existing regulatory requirements, which are announced annually by the supervisory authority. However, due to the unavailability of the insurer's loss data, this method cannot be applied in the present study.

A second and widely accepted approach is based on the mean excess plot, which is regarded as one of the most robust diagnostic tools in Extreme Value Theory for modeling excess losses in earthquake data using the GPD. The mean excess plot is constructed for different values of *D*, representing the number of fatalities. A threshold beyond which the plot exhibits approximately linear behavior is considered a suitable candidate.

As illustrated in Figure 3, the mean excess plot appears to flatten at values of 3,000, 12,000, and 31,000 fatalities. Accordingly, a threshold of 32,000 is initially suggested. Nevertheless, this threshold is impractical for reinsurance applications, as the empirical data indicate that the number of fatalities exceeds this level in only a single observation.

Another method for determining the threshold is based on the return period criterion. Under this approach, the insurer retains high-frequency, low-severity losses and transfers low-frequency, and high-severity losses to the reinsurer. For example, the primary insurer may decide to retain claims associated with a return period shorter than *T*, while ceding losses with longer return periods to reinsurance coverage.

An alternative approach, proposed by Riedman (2018) and adopted in this study, is the percentage-based method, which determines the threshold as a fixed percentage of candidate threshold values. An analysis of the earthquake dataset indicates that approximately 73% of events are classified as minor.

As a preliminary step, the kurtosis of the observed data is examined to assess whether the distribution exhibits heavy-tailed behavior, with particular attention to whether the kurtosis exceeds three. The percentage-based method is then employed to determine an appropriate threshold value. Extreme observations are subsequently defined as those data points that exceed the selected threshold.

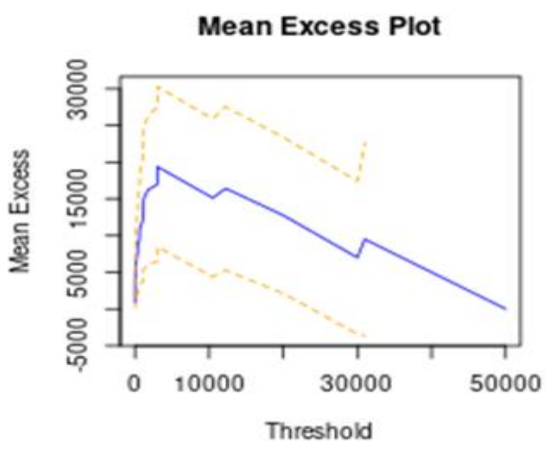

Figure 3. Mean excess plot of deaths

According to Rydman (2018), 27% of the data are extreme values. The steps of the percentage method (27%) are as follows:

Step 1. Sort data from largest to smallest.

Step 2. Calculate the number of extreme data with the following equation

$$K=27\%\times N,$$

Where K and N are the number of extreme data and the number of samples, respectively.

Step 3. Determine the threshold value with the following equation.

$$U=K+1. \qquad (4)$$

Based on equation (4), the number of extreme data is 51. Therefore, the threshold value is the 52^{nd} data which is 12. Given the substantial gap between 12 and 50,000, assuming a reinsurance ceiling of 50,000 would result in a prohibitively high reinsurance premium. Such a structure would effectively leave the primary insurer with no meaningful retention of risk. Using Equation (3), a layer limit of *L*=11,466 is obtained. An examination of the dataset indicates that the number of fatalities exceeds this ceiling in only four earthquake events. As the majority of loss events fall within this layer, the corresponding reinsurance premium is expected to be substantial. Super catastrophic events may therefore be treated as force majeure occurrences, which in practice necessitate government intervention.

To examine the impact of the threshold value on the reinsurance premium, several percentage-based threshold selection scenarios are considered. The corresponding results are reported in Table 2. As

shown therein, the threshold value plays a crucial role in determining the reinsurance premium P.
Once an appropriate threshold has been identified, the Generalized Pareto Distribution parameters are estimated using the maximum likelihood estimation (MLE) method. The estimated shape and scale parameters are 0.4823 and 2715.93, respectively.

The Kolmogorov–Smirnov test is employed to conduct the following hypothesis test.

H_0= the data has been distributed according to a generalized Pareto distribution.

The p-value of the test is 0.82032. Accordingly, the null hypothesis cannot be rejected at the 5% significance level.
Given that 187 hazardous events are observed over a 73-year period, the event frequency is modeled by a Poisson distribution with mean $\lambda = 2.56\ (= \frac{187}{73})$. The parameter values are set to $\rho = 0.3$, c=10,000,000 IRR, retention level (number of deaths) is d=5, 7, 12 and quota-share factor $\alpha = 0.20, 0.40, 0.60$. The resulting CAT reinsurance premium is reported in Table 2.

Table 2. Premium calculated under QS-XL hybrid reinsurance model

α	d	P (million IRR)
0.2	5	179,167,086
0.4	5	358,334,171
0.6	5	537,501,257
0.2	7	153,571,788
0.4	7	307,143,575
0.6	7	460,715,363
0.2	12	138,214,609
0.4	12	276,429,218
0.6	12	414,643,827

Table 2 indicates that, as the retention share d increases, the reinsurance premium decreases, reflecting the larger proportion of losses retained by the primary insurer and the corresponding reduction in the size of the reinsurance layer. Conversely, the premium increases linearly with respect to the loading parameter α. For a fixed value of α, the following relationship is observed:

$$premium_{d=5} > premium_{d=7} > premium_{d=12}$$

Holding the retention level d constant, the results further show that:

$$premium_{\alpha=0.2} > premium_{\alpha=0.4} > premium_{\alpha=0.6}$$

This behavior confirms that an increase in the reinsurer's share of the loss leads to a higher reinsurance premium. Based on these results, the ceding company and the reinsurer may jointly select the threshold level that is most appropriate for determining the reinsurance premium.
Figure 4 illustrates the evolution of the reinsurance premium under varying values of the QS coefficient α and different retention levels d. The curves depict how the premium responds to increases in α across alternative values of d.

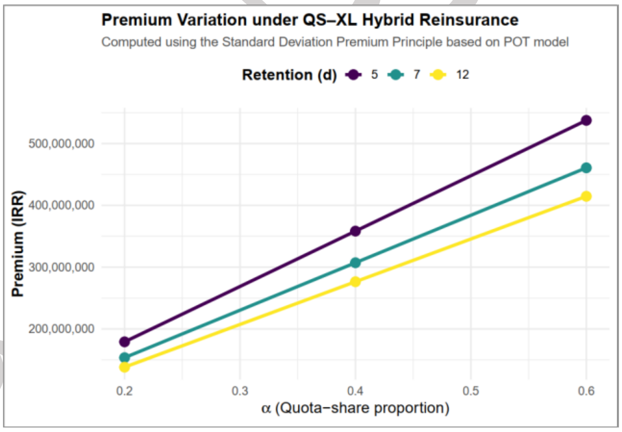

Figure 4. Premium variation under QS-XL hybrid reinsurance

6. Conclusions

The distinctly heavy-tailed nature of catastrophe loss data substantially constrains reinsurers' willingness to underwrite direct catastrophe layers. To promote underwriting participation and ensure actuarially sound pricing, the premium calculation framework for such contracts adopts the Standard Deviation Premium Principle rather than the traditional net premium rule, as it explicitly accounts for the heightened uncertainty associated with extreme loss events.
In this study, the POT model is employed to fit empirical earthquake loss data, allowing for an accurate characterization of tail behavior. Based on the fitted model, the parameters of the Generalized Pareto Distribution are estimated using Maximum Likelihood Estimation. Reinsurance premiums are then computed

within a hybrid QS–XL structure across various combinations of quota-share proportions α and retention levels d.

The empirical results demonstrate that reinsurance premiums are inversely related to the retention level d. Specifically, higher retention by the primary insurer reduces the size of the reinsurance layer, leading to lower premiums. Conversely, premiums increase linearly with respect to the α, reflecting the reinsurer's growing share of losses. These monotonic relationships confirm the strong sensitivity of catastrophe reinsurance pricing to contractual design parameters.

Furthermore, a comprehensive sensitivity analysis reveals that premium estimates are highly responsive to threshold selection within the POT framework. Higher threshold percentiles consistently generate higher reinsurance premiums, highlighting the trade-off between underwriting capacity and loss severity in catastrophe risk transfer. Consequently, the choice of an appropriate threshold emerges as a critical negotiation parameter between ceding insurers and reinsurers, ensuring that premiums remain both statistically coherent and commercially viable.

Future research may extend the proposed framework by incorporating clustered extreme events or serial dependence structures, which give rise to multiple large losses within a single portfolio year, thereby further enhancing the robustness and realism of QS–XL catastrophe risk pricing models.

Contribution of each author

First author: Writing – original draft, Data gathering, Methodology, Data analysis, Conceptualization;

Second author: Writing – review & editing, Methodology, Conceptualization.

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Conflict of interest

The authors declare that there is no conflict of interest regarding the publication of this manuscript. All ethical standards, including the avoidance of plagiarism, data falsification, misconduct, duplicate publication, and redundant submission, have been fully observed by the authors

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